

On Applying Point-Interval Logic to Criminal Forensics

(Student Paper)

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**Command and Control Research and Technology
Symposium**

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- **Formal Logic**
- **A Logic for Time**
- **Point-Interval Logic**
- **Point Graphs**
- **Temper – Software Implementation of Point-Interval Formalism**
- **Applying Point-Interval Logic to London Bombing Data**

- Logic is the “Art of Reasoning”.
- Logic is used to make inferences based on the available information.
- Formal logic makes inferences based purely on the form of the content, without any understanding of the meaning of the content.
- Reasoning based just on the form is important because this means computers can do it.

All humans are mortal.

Socrates is a human.

Therefore Socrates is mortal.

$$\forall x \ x \in h \rightarrow x \in m$$

$$s \in h$$

?

A logic for time will enable us to:

- **Characterize time-sensitive attributes of a domain to be modeled**
- **Do temporal analysis of a domain, which will help us in developing a better understanding of the relationship between domain entities**
- **To identify Inconsistencies and anomalies**

There are very few approaches that allow explicit representation of time and reasoning about it

Allen's Interval Logic



- Allen introduced Interval Algebra as a framework for temporal reasoning. The algebra takes time intervals to be primitives. There are 13 possible relationships between a pair of intervals:


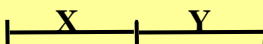

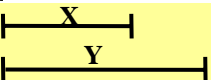
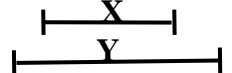
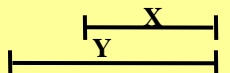
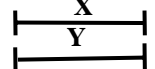
$R = \{\text{before, meets, overlaps, starts, during, finishes, equals, after, met-by, started by, contains, finished by}\}$

- A temporal relation is represented as ' $X_i C_{ij} X_j$ ', where $C_{ij} \subseteq R$, and X_i and X_j are intervals. The elements in the set C_{ij} form a disjunctive temporal constraint on relationships between the two intervals.
- The problem of determining consistency in Allen's Interval Algebra is NP Complete.

Point-Interval Logic (PIL) is a Pointisable logic. It is a tractable subclass of Allen's interval logic.

Case I: X and Y both intervals with non-zero lengths

$X = [sx, ex]$, $Y = [sy, ey]$ with $sx < ex$ and $sy < ey$

Before	X < Y	$ex < sy$	
Meets	X m Y	$ex = sy$	
Overlaps	X o Y	$sx < sy; sy < ex; ex < ey$	
Starts	X s Y	$sx = sy; ex < ey$	
During	X d Y	$sx > sy; ex < ey$	
Finishes	X f Y	$sy < sx; ey = ex$	
Equals	X = Y	$sx = sy; ex = ey$	

Point-Interval Logic



Case II: X and Y both points

X = [px] and Y = [py] with $sx = ex = px$ and $sy = ey = py$

Before	X < Y	$px < py$	$\overset{X}{\underset{\bullet}{px}}$ $\overset{Y}{\underset{\bullet}{py}}$
Equals	X = Y	$px = py$	$\overset{[X;Y]}{\underset{\bullet}{px}}$

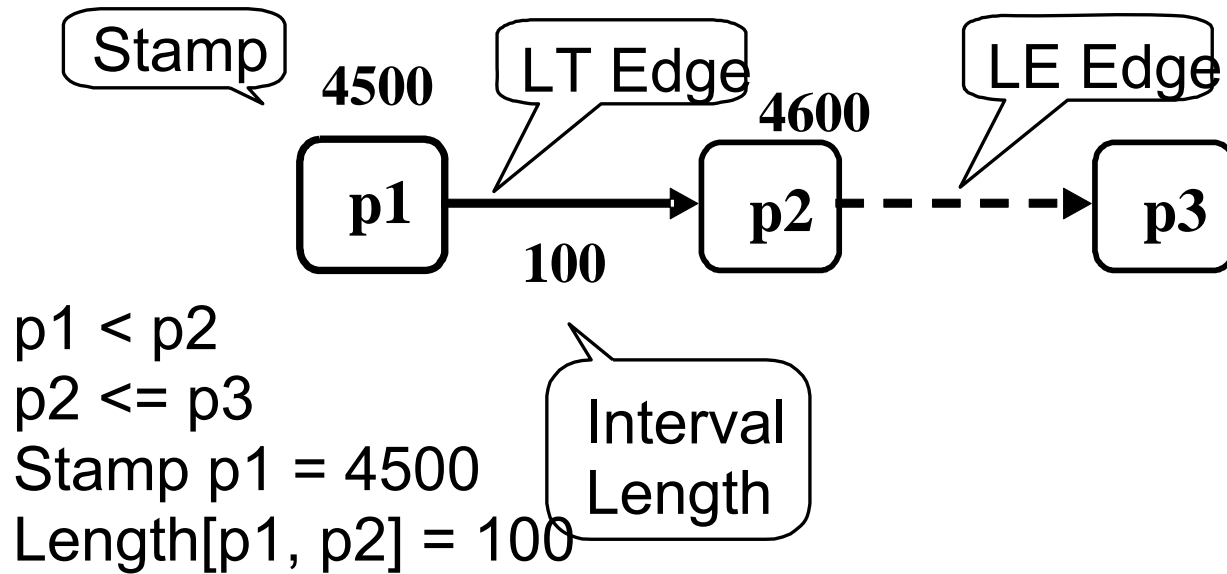
A point-point relation “less than or equal to” (\leq) can be added to PIL without losing tractability.

Case III—X is a point and Y is an interval: X = [px] and Y = [sy, ey] with $px = sx = ex$ and $sy < ey$

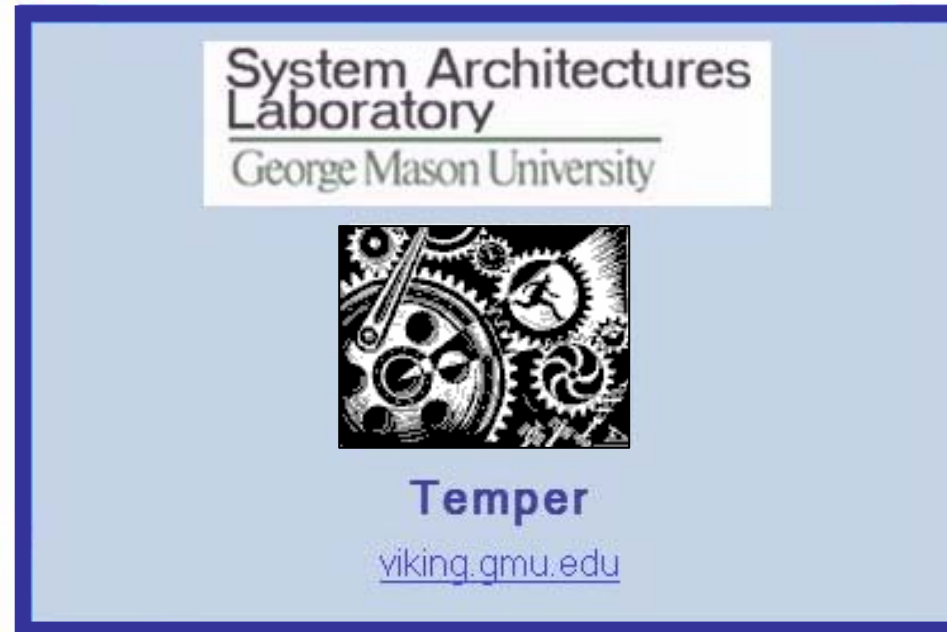
Before	X < Y	$px < sy$	$\overset{X}{\bullet}$ $\overline{\hspace{1cm}}^Y$
Starts	X s Y	$px = sy$	$\overset{X}{\bullet} \overline{\hspace{1cm}}^Y$
During	X d Y	$sy < px < ey$	$\overline{\hspace{1cm}}^X \overline{\hspace{1cm}}^Y$
Finishes	X f Y	$px = ey$	$\overline{\hspace{1cm}}^Y \overset{X}{\bullet}$
Before	Y < X	$ey < px$	$\overline{\hspace{1cm}}^Y$ $\overset{X}{\bullet}$

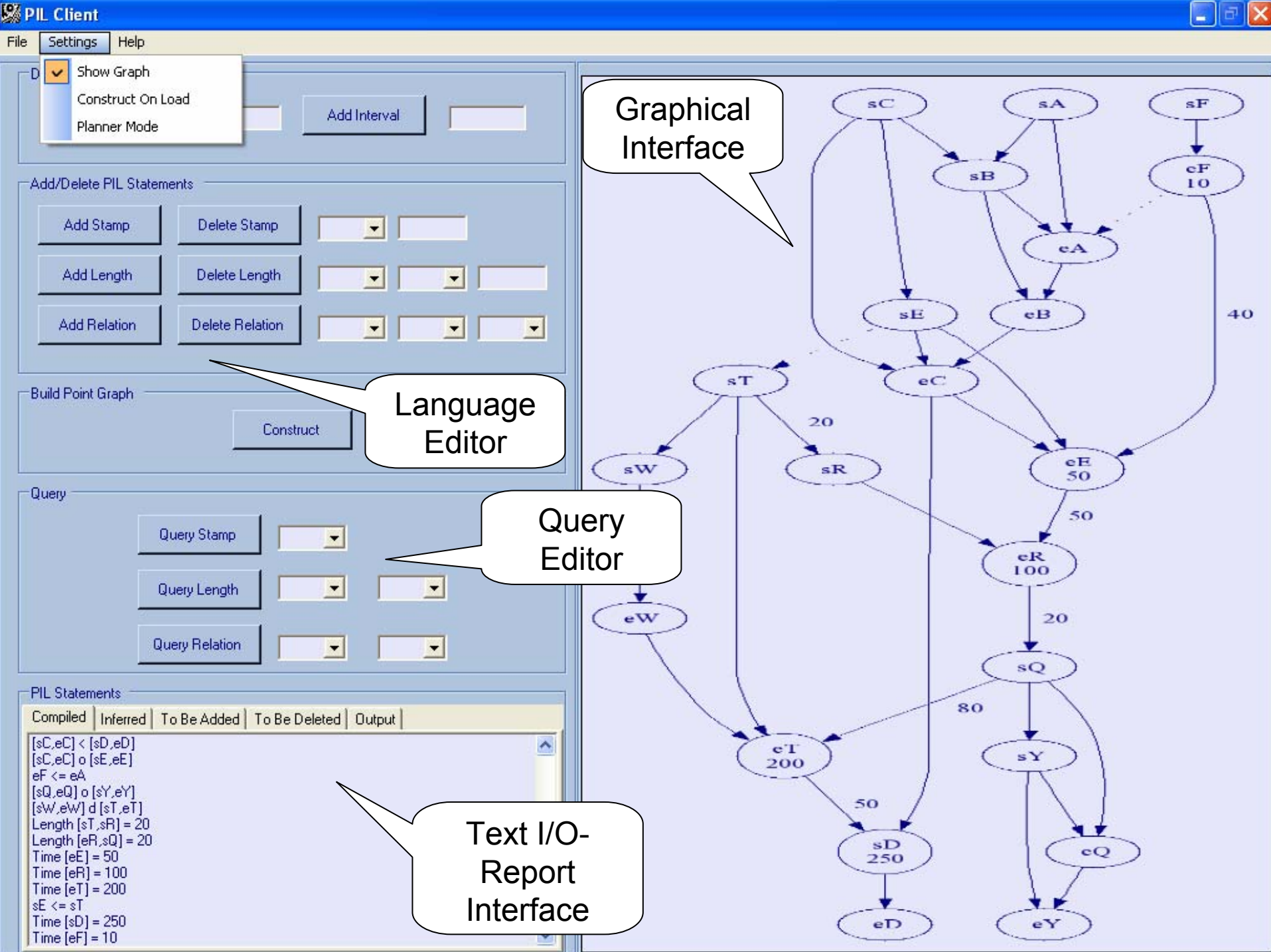
- Quantitative Temporal Information
 - $d1 \leq \text{Length } [X, Y] \leq d2$
 - $t1 \leq \text{Stamp } [X] \leq t2$where $d1$, $d2$, $t1$, and $t2$ are rational numbers, and X , Y are points

Point Graphs



PIL Statements and the corresponding Point Graph

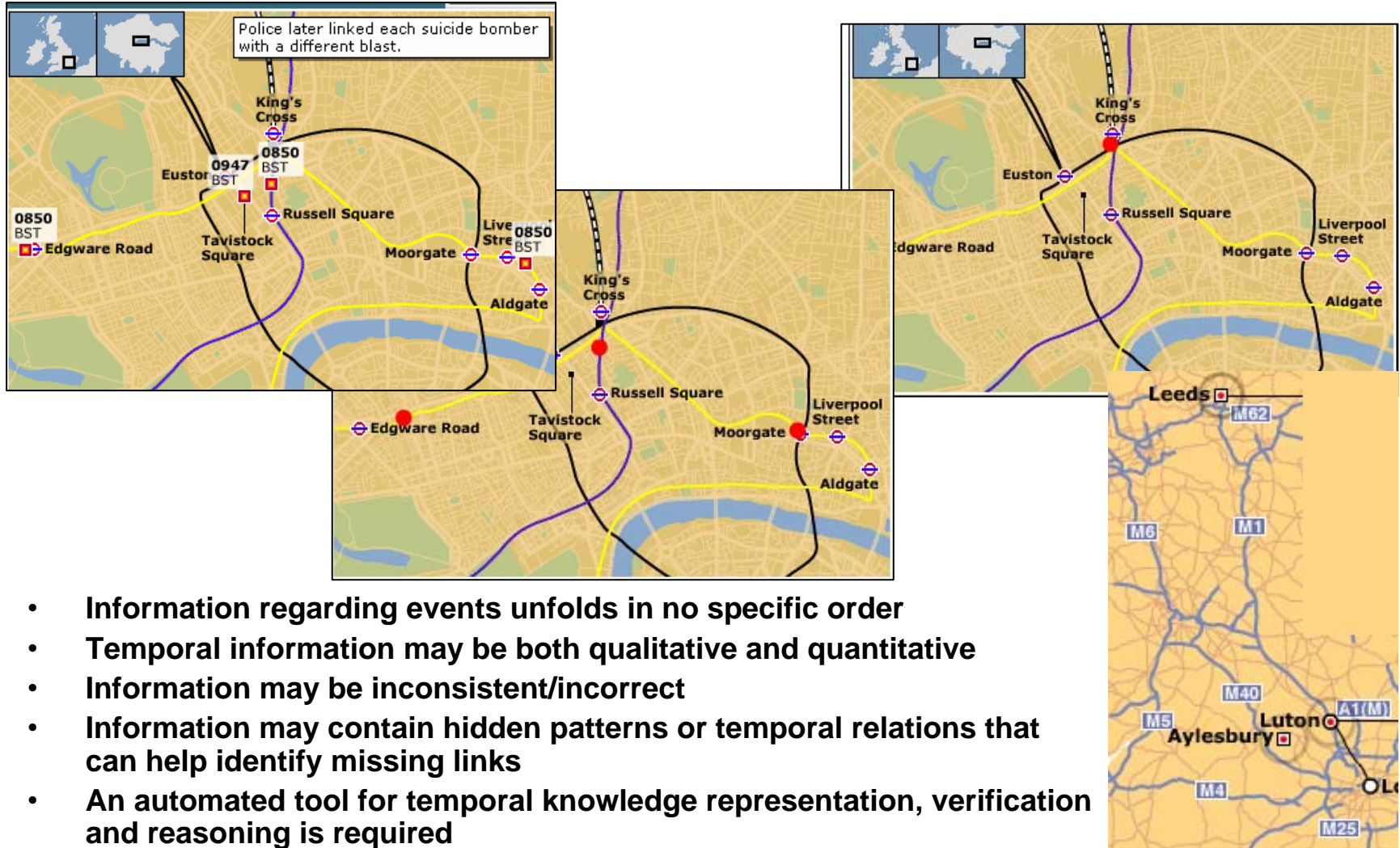




Modeling with Temper



- Convert the available temporal information into statements in Point-Interval Logic.
- Input these statements to Temper using the language editor of Temper.
- Construct a Point Graph representation of the set of PIL statements.
- If the set of PIL statements is inconsistent then Temper will not be able to construct the Point Graph representation.
- Temper will identify the subset of PIL statements causing the inconsistency.
- Remove the inconsistent statements.
- Once a consistent Point Graph has been constructed, it can be used to draw inferences.

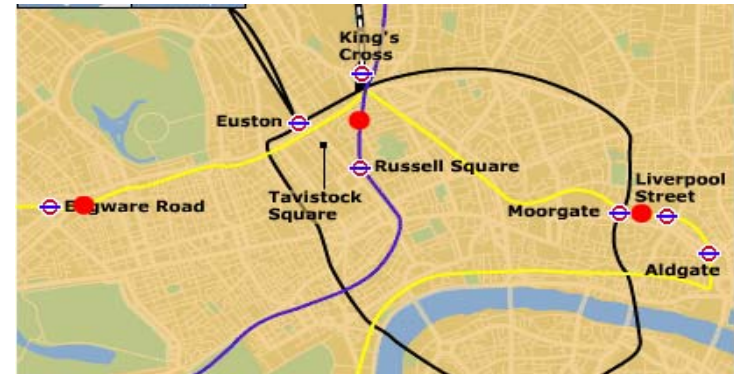


- Information regarding events unfolds in no specific order
- Temporal information may be both qualitative and quantitative
- Information may be inconsistent/incorrect
- Information may contain hidden patterns or temporal relations that can help identify missing links
- An automated tool for temporal knowledge representation, verification and reasoning is required

Example: London Bombing



- There were four explosions in London.
- The sites of these explosions were: Travistock Square, Edgware Road, Aldgate and Russell Square.
- Three of these explosions (Edgware, Aldgate and Russell Square) were in trains.
- These trains left from King's Cross station. The journey of these trains ended in explosions.
- The time it takes a train from King's Cross to reach Edgware is at least 5 time units.
- The time it takes a train from King's Cross to reach Aldgate is at least 4 time units.
- The time it takes a train from King's Cross to reach Russell Square is at least 5 time units.



Interval Train_King_Cross_to_Edgware,
Train_King_Cross_to_Aldgate,
Train_King_Cross_to_Russell_Sq

Point Explosion_at_Travistock_Square,
Explosion_near_Edgware,
Explosion_near_Aldgate,
Explosion_near_Russell_Sq

Explosion_near_Edgware *finish*
Train_King_Cross_to_Edgware

Explosion_near_Aldgate *finish*
Train_King_Cross_to_Aldgate

Explosion_near_Russell_Sq *finish*
Train_King_Cross_to_Russell_Sq

Length [Train_King_Cross_to_Edgware] ≥ 5

Length [Train_King_Cross_to_Aldgate] ≥ 4

Length [Train_King_Cross_to_Russell_Sq] ≥ 5

Declare Variables

Add Point

Add Interval

Add/Delete PIL Statements

Add Stamp

Delete Stamp

Add Length

Delete Length

Add Relation

Delete Relation

Add Composite Relation

<
m
o
s

Build Point Graph

Query

Query Stamp

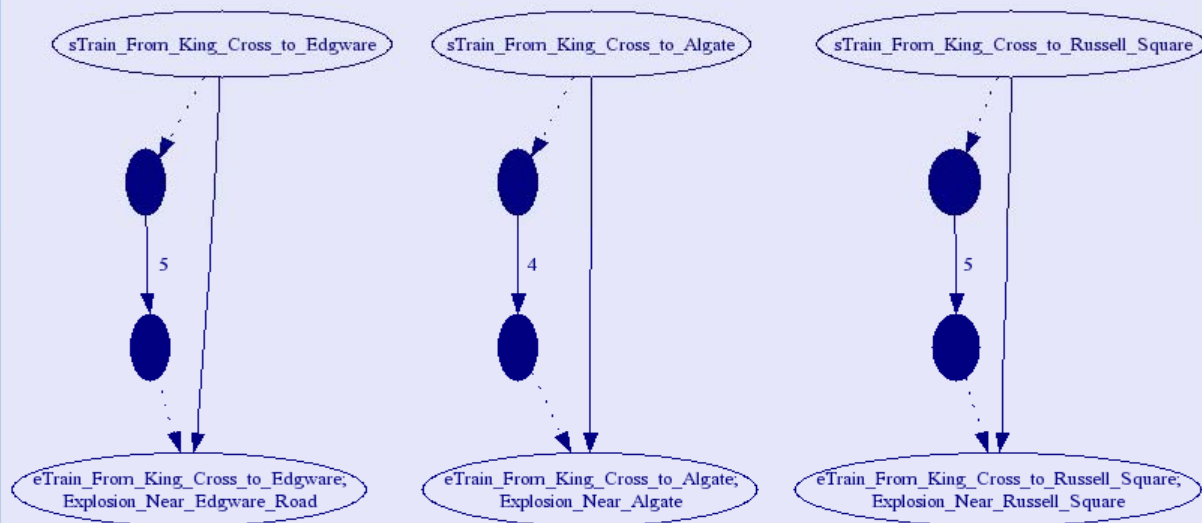
Query Length

Query Relation

PIL Statements

Compiled Inferred To Be Added To Be Deleted Output CB

```
sTrain_From_King_Cross_to_Edgware < eTrain_From_King_Cross_to_Edgware  
sTrain_From_King_Cross_to_Algate < eTrain_From_King_Cross_to_Algate  
sTrain_From_King_Cross_to_Russell_Square < eTrain_From_King_Cross_to_Russell_Square  
Explosion_Near_Edgware_Road f [sTrain_From_King_Cross_to_Edgware, eTrain_From_King_Cross_to_Edgware]  
Explosion_Near_Russell_Square f [sTrain_From_King_Cross_to_Russell_Square, eTrain_From_King_Cross_to_Russell_Square]  
Explosion_Near_Algate f [sTrain_From_King_Cross_to_Algate, eTrain_From_King_Cross_to_Algate]  
Length [sTrain_From_King_Cross_to_Edgware, eTrain_From_King_Cross_to_Edgware] >= 4  
Length [sTrain_From_King_Cross_to_Algate, eTrain_From_King_Cross_to_Algate] >= 4  
Length [sTrain_From_King_Cross_to_Russell_Square, eTrain_From_King_Cross_to_Russell_Square] >= 4
```

Point
GraphPIL
statements

Declare Variables

Add Point

Add Interval

Add/Delete PIL Statements

Add Stamp

Delete Stamp

Add Length

Delete Length

Add Relation

Delete Relation

Add Composite Relation

<

m

o

s

Build Point Graph

Construct

Query

Query Stamp

sTrain

Query Length

Query Relation

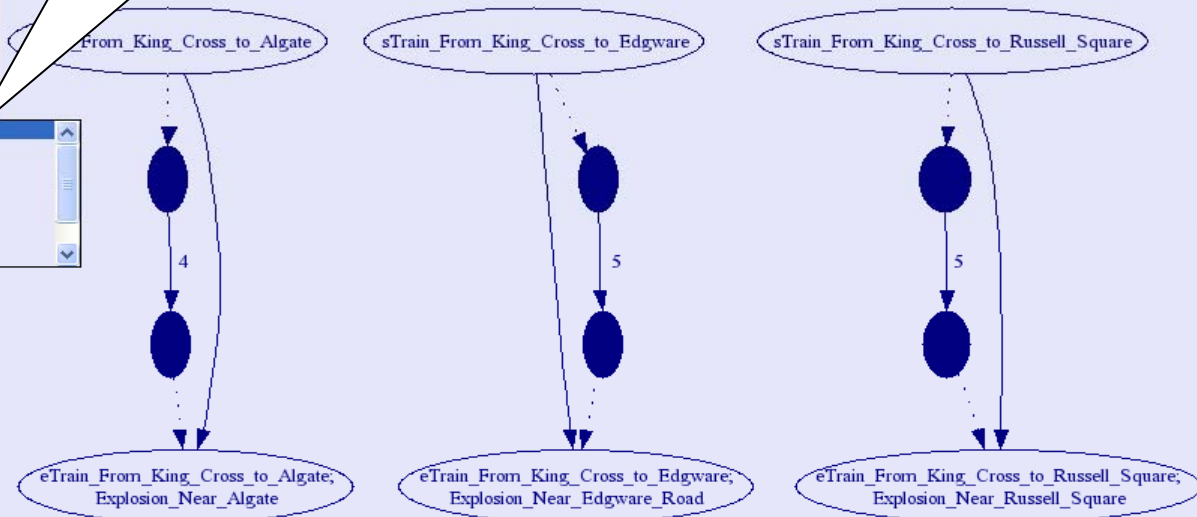
sTrain_From_King_Cross_to_Edgware
 eTrain_From_King_Cross_to_Edgware
 sTrain_From_King_Cross_to_Algate
 eTrain_From_King_Cross_to_Algate
 sTrain_From_King_Cross_to_Russell_Square
 eTrain_From_King_Cross_to_Russell_Square
 Explosion_At_Tavistock_Square
 Explosion_Near_Edgware_Road

PIL Statements

Compiled Inferred To Be Added To Be Deleted Output CBP

```
sTrain_From_King_Cross_to_Edgware < eTrain_From_King_Cross_to_Edgware
sTrain_From_King_Cross_to_Algate < eTrain_From_King_Cross_to_Algate
sTrain_From_King_Cross_to_Russell_Square < eTrain_From_King_Cross_to_Russell_Square
Explosion_Near_Edgware_Road f [sTrain_From_King_Cross_to_Edgware, eTrain_From_King_Cross_to_Edgware]
Explosion_Near_Russell_Square f [sTrain_From_King_Cross_to_Russell_Square, eTrain_From_King_Cross_to_Russell_Square]
Explosion_Near_Algate f [sTrain_From_King_Cross_to_Algate, eTrain_From_King_Cross_to_Algate]
Length [sTrain_From_King_Cross_to_Edgware, eTrain_From_King_Cross_to_Edgware] >= 4
Length [sTrain_From_King_Cross_to_Algate, eTrain_From_King_Cross_to_Algate] >= 4
Length [sTrain_From_King_Cross_to_Russell_Square, eTrain_From_King_Cross_to_Russell_Square] >= 4
```

query Stamp (when did the train to Edgware leave from King's Cross?)



Temper

File Settings Help

Declare Variables

Add Point

Add Interval

Add/Delete PIL Statements

Add Stamp

Delete Stamp

=

Add Length

Delete Length

=

Add Relation

Delete Relation

Add Composite Relation

<

m

o

s

Build Point Graph

Construct

Query

Query Stamp

sTrain

Query Length

Query Relation

PIL Statements

Compiled

Inferred

To Be Added

To Be Deleted

Output

CBP

sTrain_From_King_Cross_to_Edgware < eTrain_From_King_Cross_to_Edgware

sTrain_From_King_Cross_to_Algate < eTrain_From_King_Cross_to_Algate

sTrain_From_King_Cross_to_Russell_Square < eTrain_From_King_Cross_to_Russell_Square

Explosion_Near_Edgware_Road f [sTrain_From_King_Cross_to_Edgware,eTrain_From_King_Cross_to_Edgware]

Explosion_Near_Russell_Square f [sTrain_From_King_Cross_to_Russell_Square,eTrain_From_King_Cross_to_Russell_Square]

Explosion_Near_Algate f [sTrain_From_King_Cross_to_Algate,eTrain_From_King_Cross_to_Algate]

Length [sTrain_From_King_Cross_to_Edgware,eTrain_From_King_Cross_to_Edgware] >= 4

Length [sTrain_From_King_Cross_to_Algate,eTrain_From_King_Cross_to_Algate] >= 4

Length [sTrain_From_King_Cross_to_Russell_Square,eTrain_From_King_Cross_to_Russell_Square] >= 4

query Stamp (when did the train to Edgware leave from King's Cross?)

unknown

Stamp

?

OK

sTrain_From_King_Cross_to_Edgware

sTrain_From_King_Cross_to_Russell_Square

sTrain_From_King_Cross_to_Edgware

eTrain_From_King_Cross_to_Algate; Explosion_Near_Algate

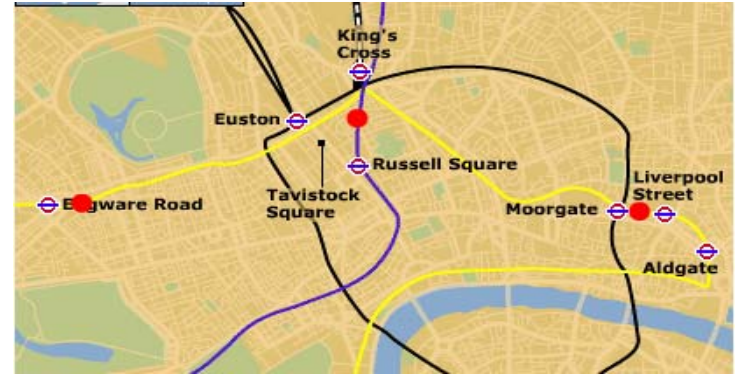
eTrain_From_King_Cross_to_Russell_Square; Explosion_Near_Russell_Square

eTrain_From_King_Cross_to_Edgware; Explosion_Near_Edgware_Road

Example: London Bombing (cont'd)



- The explosion near Edgware Road took place between time units 840 and 852.
- The explosion near Aldgate took place between time units 845 and 850.
- The explosion near Russell Square took place between time units 840 and 850.
- The explosion at Travistock Square took place between time units 945 and 955.



840 <= Stamp [Explosion_near_Edgware] <= 852
845 <= Stamp [Explosion_near_Aldgate] <= 850
840 <= Stamp [Explosion_near_Russell_Sq] <= 850
945 <= Stamp [Explosion_at_Travistock_Square] <= 955

Declare Variables

Add Point

Add/Delete PIL Statements

Add Stamp

Delete Stamp

Add Length

Delete Length

Add Relation

Delete Relation

Add Composite Relation

Build Point Graph

Construct

Query

Query Stamp

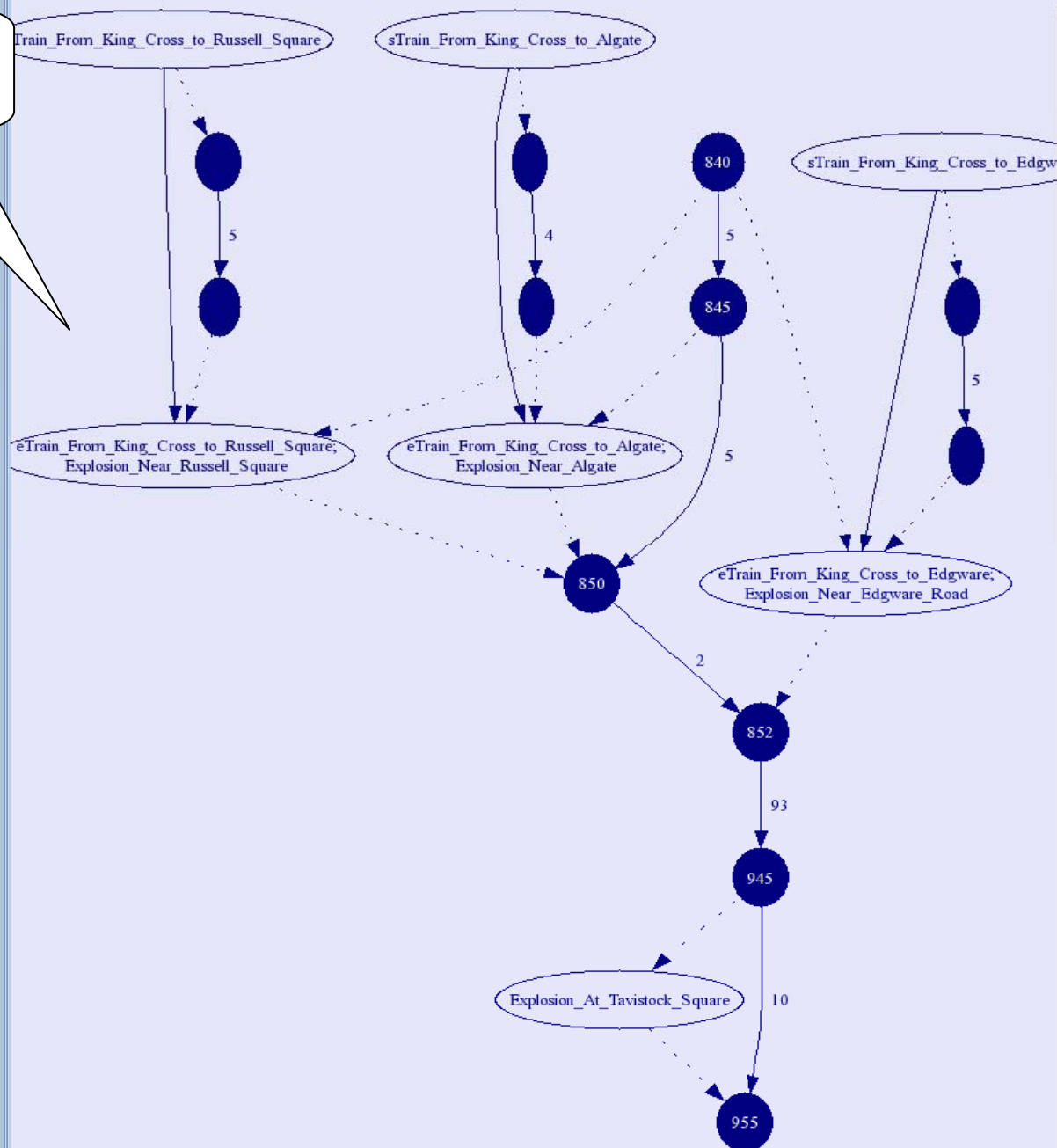
Query Length

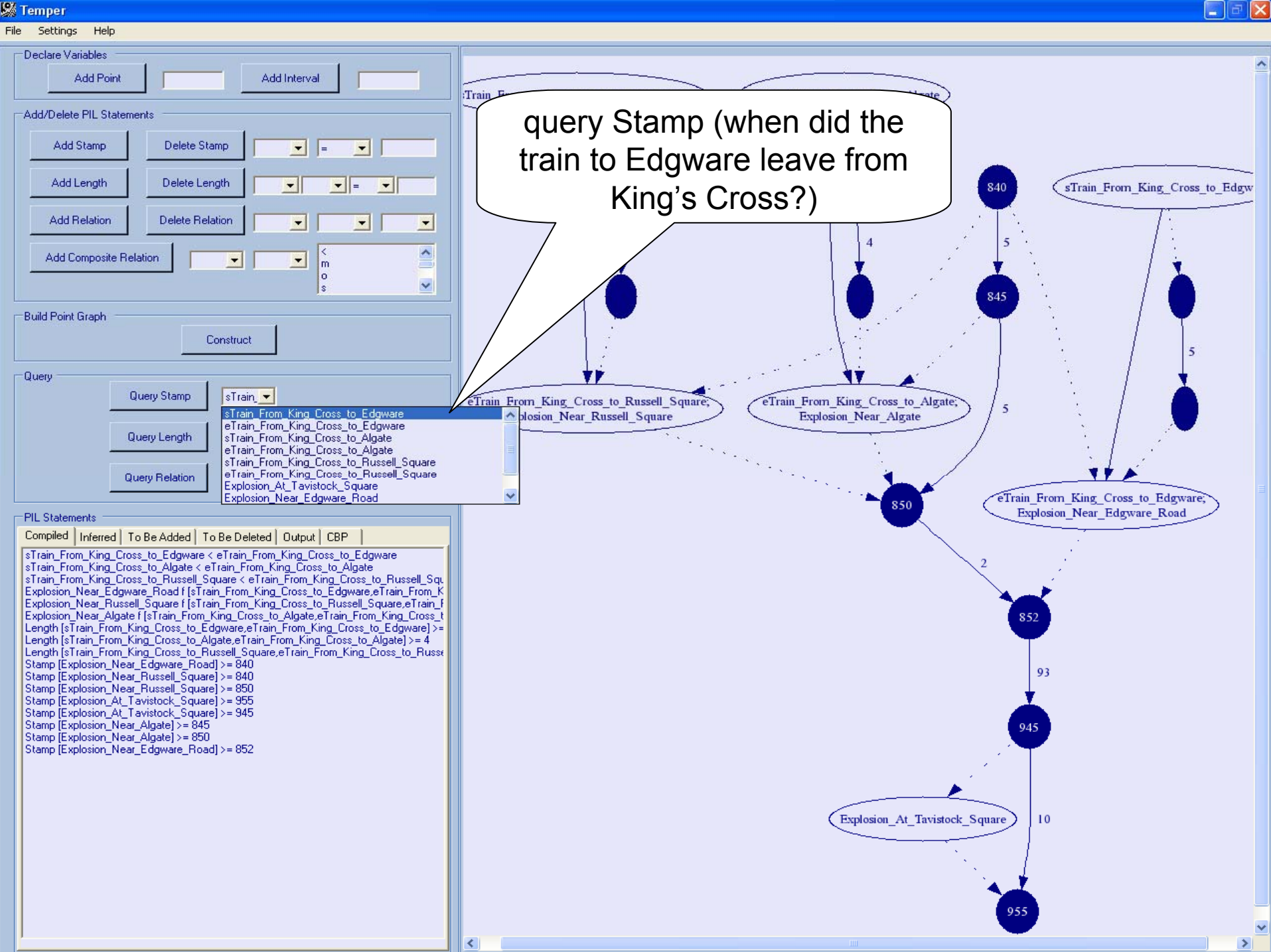
Query Relation

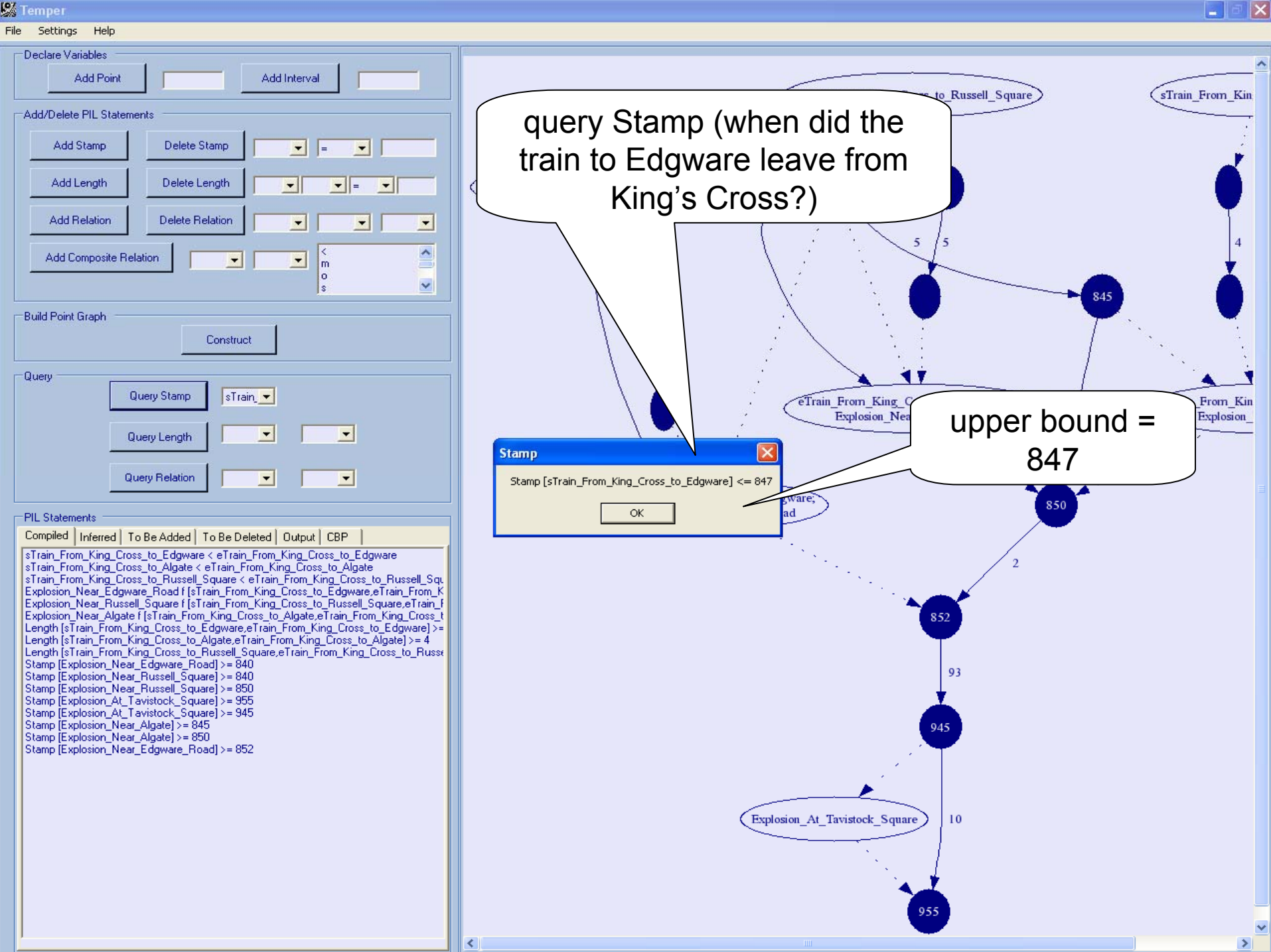
PIL Statements

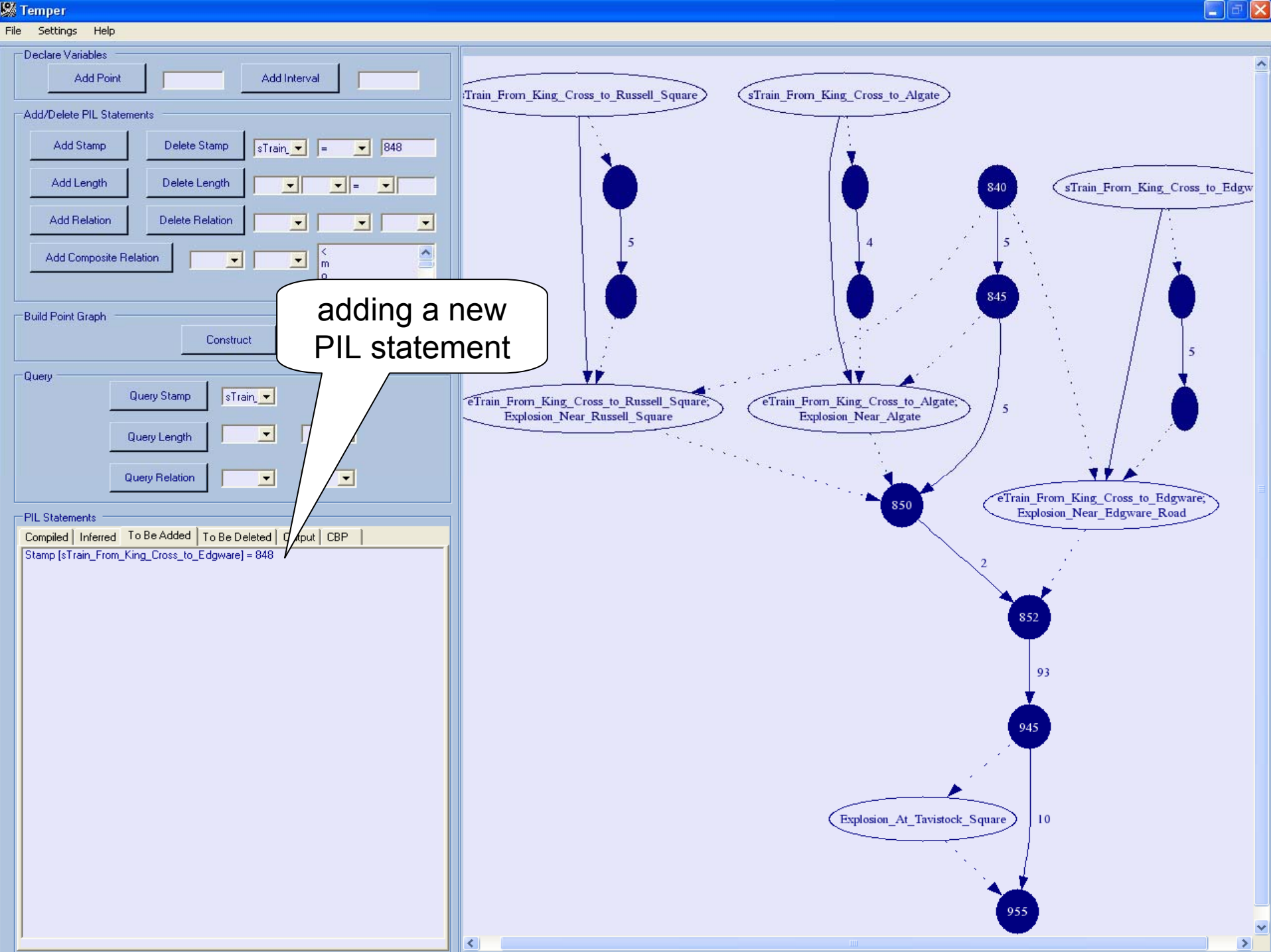
Compiled Inferred To Be Added To Be Deleted Output CBP

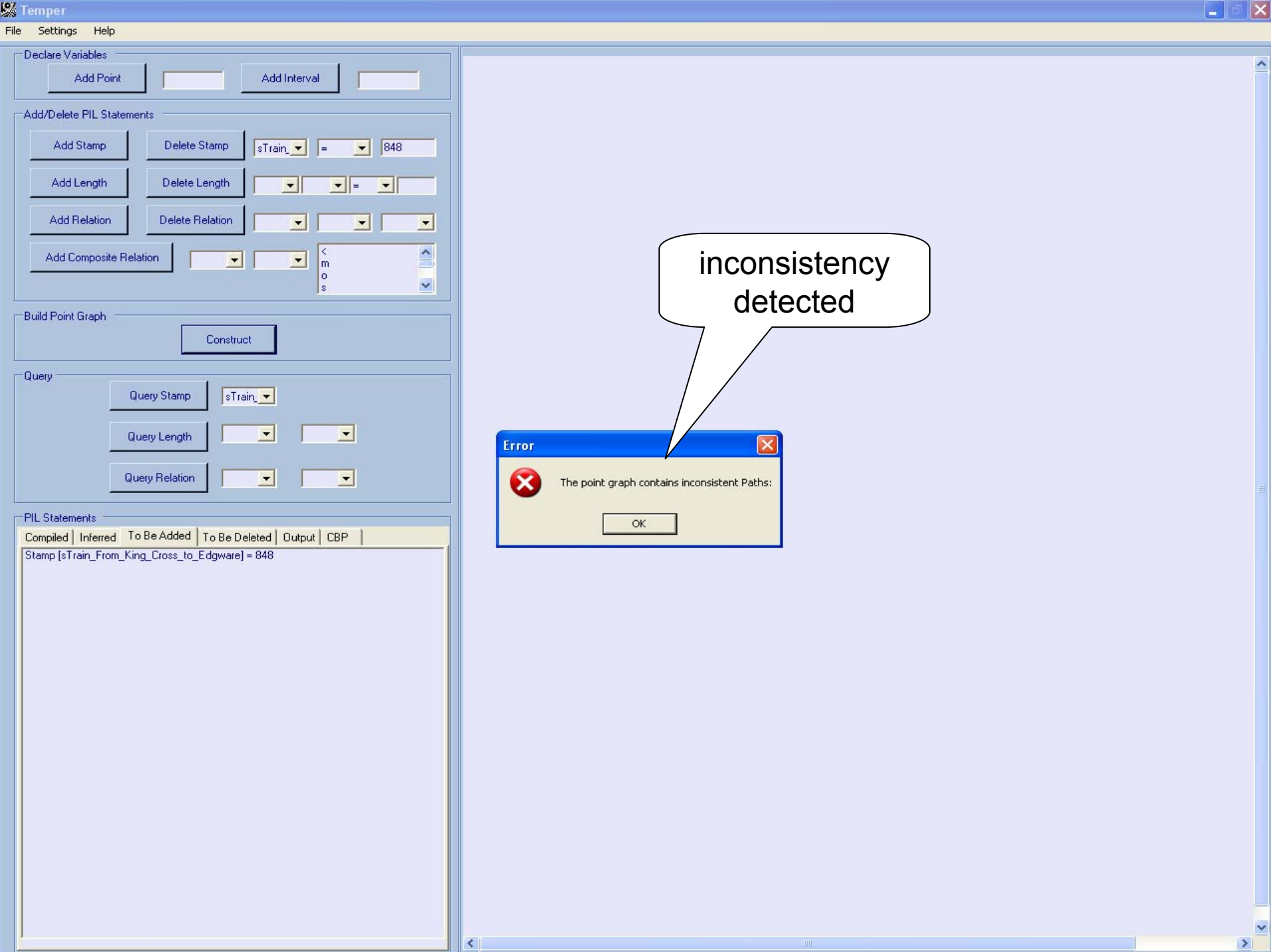
```
sTrain_From_King_Cross_to_Edgware < eTrain_From_King_Cross_to_Edgware
sTrain_From_King_Cross_to_Algate < eTrain_From_King_Cross_to_Algate
sTrain_From_King_Cross_to_Russell_Square < eTrain_From_King_Cross_to_Russell_Square
Explosion_Near_Edgware_Road f [sTrain_From_King_Cross_to_Edgware,eTrain_From_K
Explosion_Near_Russell_Square f [sTrain_From_King_Cross_to_Russell_Square,eTrain_f
Explosion_Near_Algate f [sTrain_From_King_Cross_to_Algate,eTrain_From_King_Cross_t
Length [sTrain_From_King_Cross_to_Edgware,eTrain_From_King_Cross_to_Edgware] >=
Length [sTrain_From_King_Cross_to_Algate,eTrain_From_King_Cross_to_Algate] >= 4
Length [sTrain_From_King_Cross_to_Russell_Square,eTrain_From_King_Cross_to_Russe
Stamp [Explosion_Near_Edgware_Road] >= 840
Stamp [Explosion_Near_Russell_Square] >= 840
Stamp [Explosion_Near_Russell_Square] >= 850
Stamp [Explosion_At_Tavistock_Square] >= 955
Stamp [Explosion_At_Tavistock_Square] >= 945
Stamp [Explosion_Near_Algate] >= 845
Stamp [Explosion_Near_Algate] >= 850
Stamp [Explosion_Near_Edgware_Road] >= 852
```

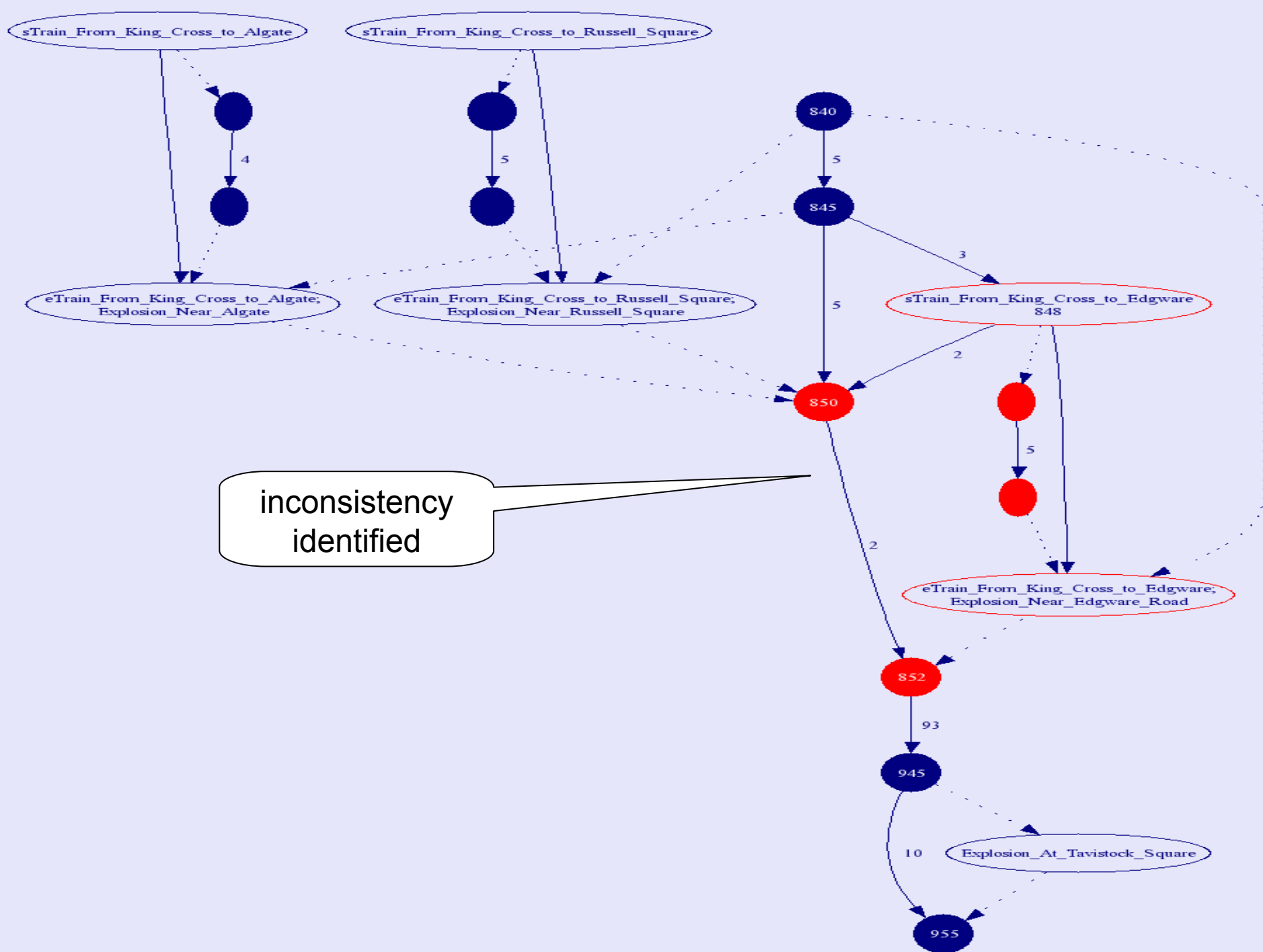
revised Point
Graph

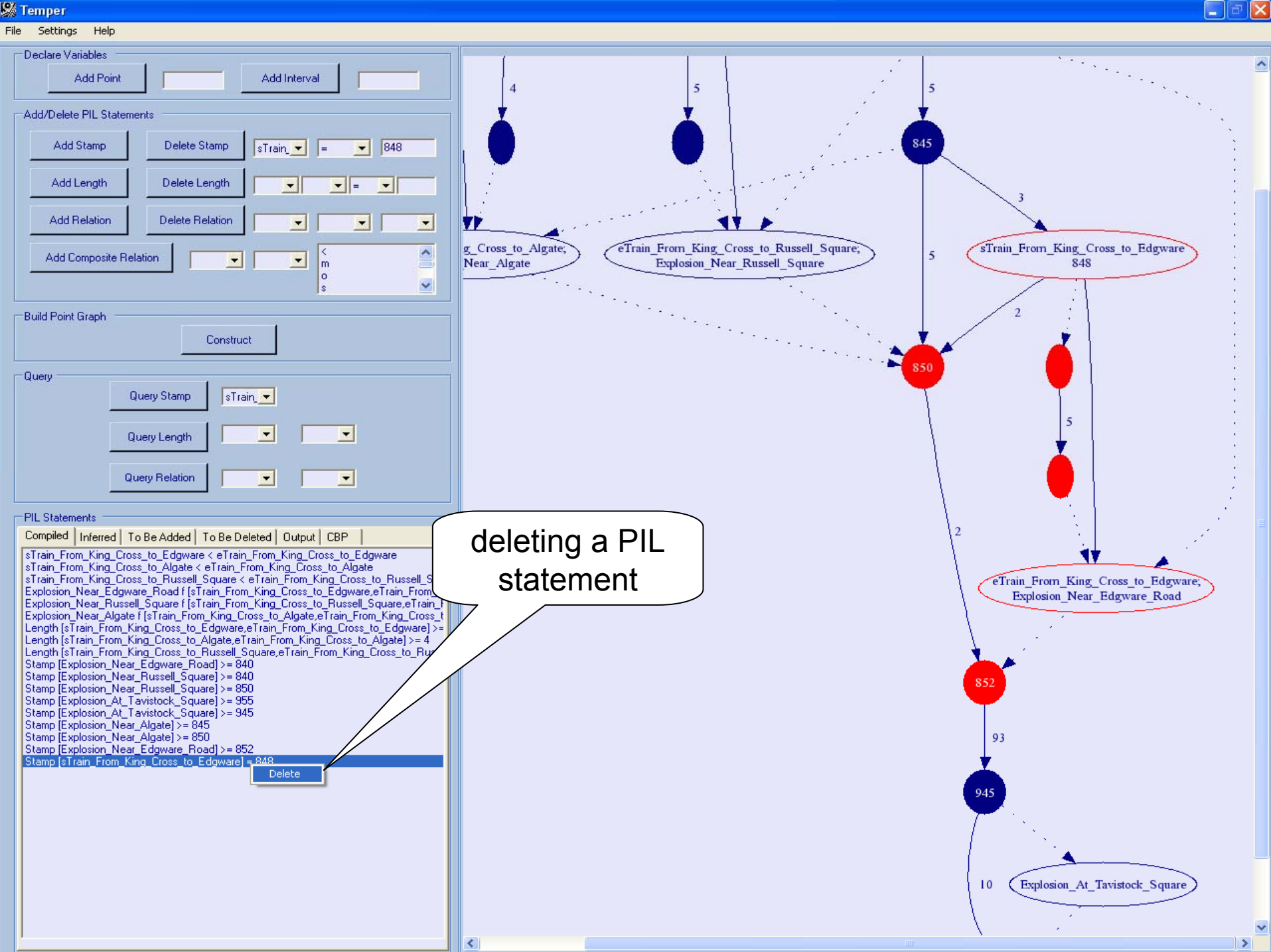












Declare Variables

Add Point

Add Interval

Add/Delete PIL Statements

Add Stamp

Delete Stamp

=

=

Add Length

Delete Length

=

=

Add Relation

Add Composite Relation

Build Point Graph

Query

Query Stamp

Query Length

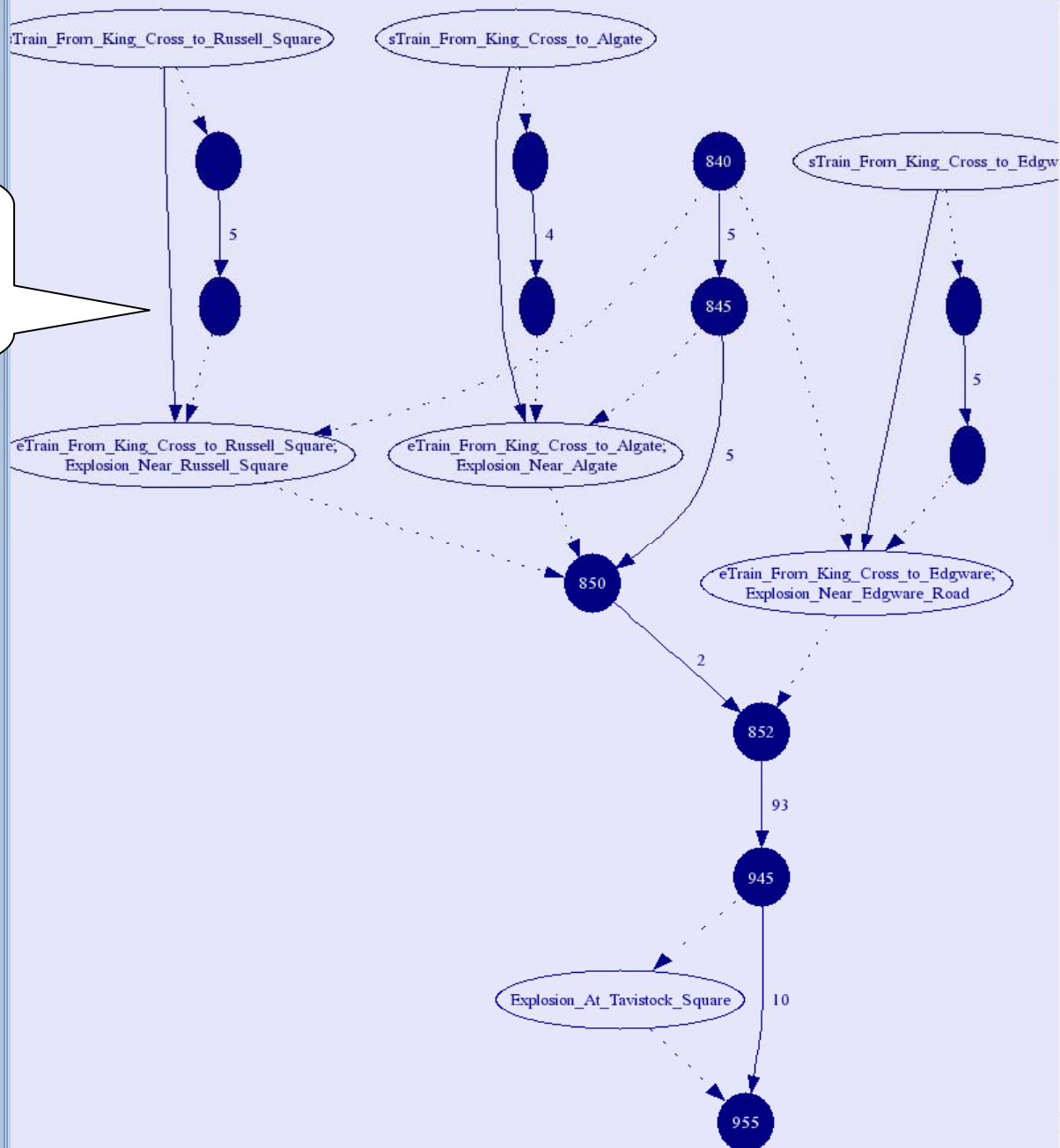
Query Relation

PIL Statements

Compiled Inferred To Be Added To Be Deleted Output CBP

```
sTrain_From_King_Cross_to_Edgware < eTrain_From_King_Cross_to_Edgware
sTrain_From_King_Cross_to_Algate < eTrain_From_King_Cross_to_Algate
sTrain_From_King_Cross_to_Russell_Square < eTrain_From_King_Cross_to_Russell_Square
Explosion_Near_Edgware_Road f [sTrain_From_King_Cross_to_Edgware,eTrain_From_K
Explosion_Near_Russell_Square f [sTrain_From_King_Cross_to_Russell_Square,eTrain_f
Explosion_Near_Algate f [sTrain_From_King_Cross_to_Algate,eTrain_From_King_Cross_t
Length [sTrain_From_King_Cross_to_Edgware,eTrain_From_King_Cross_to_Edgware] >=
Length [sTrain_From_King_Cross_to_Algate,eTrain_From_King_Cross_to_Algate] >= 4
Length [sTrain_From_King_Cross_to_Russell_Square,eTrain_From_King_Cross_to_Russe
Stamp [Explosion_Near_Edgware_Road] >= 840
Stamp [Explosion_Near_Russell_Square] >= 840
Stamp [Explosion_Near_Russell_Square] >= 850
Stamp [Explosion_At_Tavistock_Square] >= 955
Stamp [Explosion_At_Tavistock_Square] >= 945
Stamp [Explosion_Near_Algate] >= 845
Stamp [Explosion_Near_Algate] >= 850
Stamp [Explosion_Near_Edgware_Road] >= 852
```

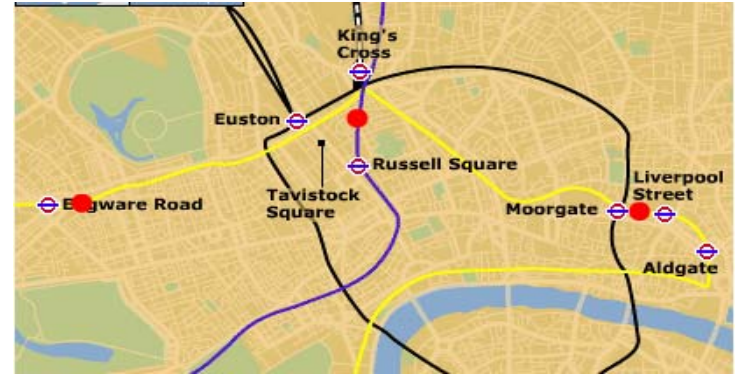
revised Point
Graph
(consistent)



Example: London Bombing (cont'd)



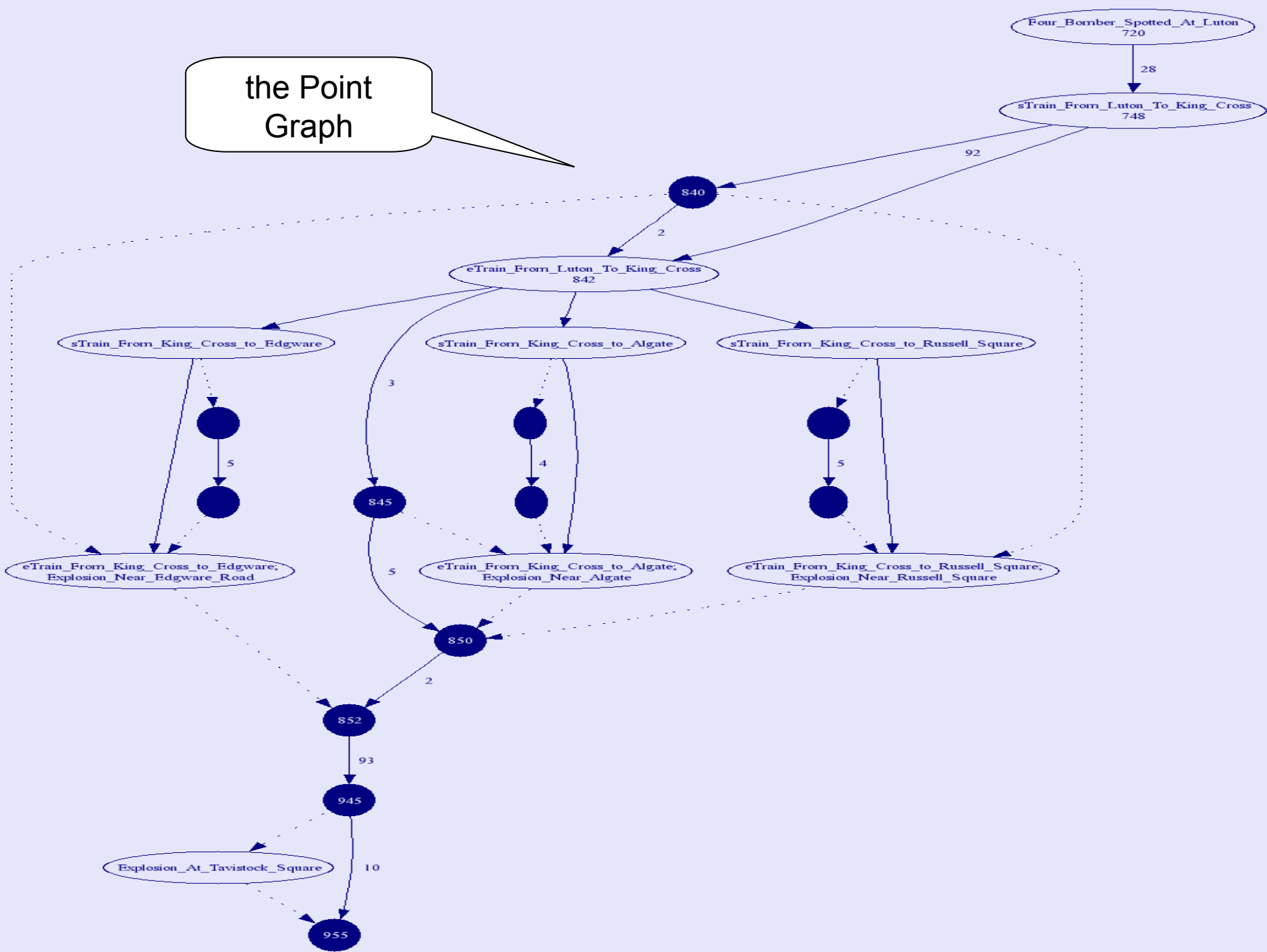
- The alleged four bombers spotted entering the Luton station at time unit 720.
- The next train from Luton to King's Cross left at 748 reaching King's Cross at 842.
- The three trains from King's Cross station in which the explosions took place, must have left King's Cross after the train from Luton reached King's Cross.

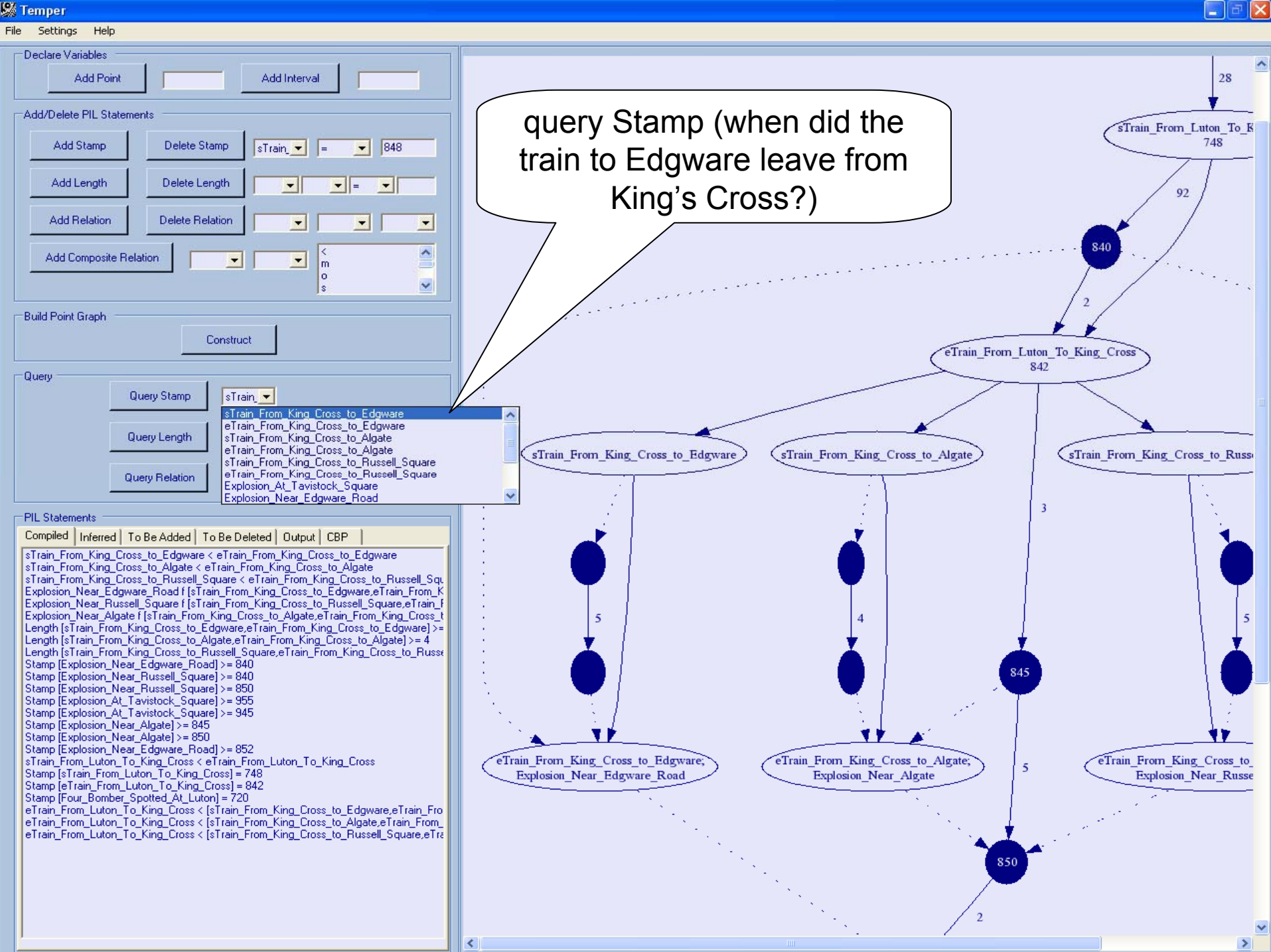


Interval Train_Luton_to_King_Cross
Point Bombers_spotted_at_Luton
Stamp [Bombers_spotted_at_Luton] = 720
Stamp [sTrain_Luton_to_King_Cross] = 748
Stamp [eTrain_Luton_to_King_Cross] = 842
eTrain_Luton_to_King_Cross less
 Train_King_Cross_to_Edgware
eTrain_Luton_to_King_Cross less
 Train_King_Cross_to_Aldgate
eTrain_Luton_to_King_Cross less
 Train_King_Cross_to_Russell_Sq



the Point Graph





Temper

File Settings Help

Declare Variables

Add Point Add Interval

Add/Delete PIL Statements

Add Stamp Delete Stamp sTrain = 848

Add Length Delete Length =

Add Relation Delete Relation

Add Composite Relation < m o s

Build Point Graph

Construct

Query

Query Stamp sTrain

Query Length

Query Relation

PIL Statements

Compiled Inferred To Be Added To Be Deleted Output CBP

```
sTrain_From_King_Cross_to_Edgware < eTrain_From_King_Cross_to_Edgware
sTrain_From_King_Cross_to_Algate < eTrain_From_King_Cross_to_Algate
sTrain_From_King_Cross_to_Russell_Square < eTrain_From_King_Cross_to_Russell_Square
Explosion_Near_Edgware_Road f [sTrain_From_King_Cross_to_Edgware,eTrain_From_King_Cross_to_Edgware]
Explosion_Near_Russell_Square f [sTrain_From_King_Cross_to_Russell_Square,eTrain_From_King_Cross_to_Russell_Square]
Explosion_Near_Algate f [sTrain_From_King_Cross_to_Algate,eTrain_From_King_Cross_to_Algate]
Length [sTrain_From_King_Cross_to_Edgware,eTrain_From_King_Cross_to_Edgware] >= 4
Length [sTrain_From_King_Cross_to_Algate,eTrain_From_King_Cross_to_Algate] >= 4
Length [sTrain_From_King_Cross_to_Russell_Square,eTrain_From_King_Cross_to_Russell_Square] >= 4
Stamp [Explosion_Near_Edgware_Road] >= 840
Stamp [Explosion_Near_Russell_Square] >= 840
Stamp [Explosion_Near_Russell_Square] >= 850
Stamp [Explosion_At_Tavistock_Square] >= 955
Stamp [Explosion_At_Tavistock_Square] >= 945
Stamp [Explosion_Near_Algate] >= 845
Stamp [Explosion_Near_Algate] >= 850
Stamp [Explosion_Near_Edgware_Road] >= 852
sTrain_From_Luton_To_King_Cross < eTrain_From_Luton_To_King_Cross
Stamp [sTrain_From_Luton_To_King_Cross] = 748
Stamp [eTrain_From_Luton_To_King_Cross] = 842
Stamp [Four_Bomber_Spotted_At_Luton] = 720
eTrain_From_Luton_To_King_Cross < [sTrain_From_King_Cross_to_Edgware,eTrain_From_King_Cross_to_Algate,eTrain_From_King_Cross_to_Russell_Square]
eTrain_From_Luton_To_King_Cross < [sTrain_From_King_Cross_to_Edgware,eTrain_From_King_Cross_to_Algate,eTrain_From_King_Cross_to_Russell_Square]
```

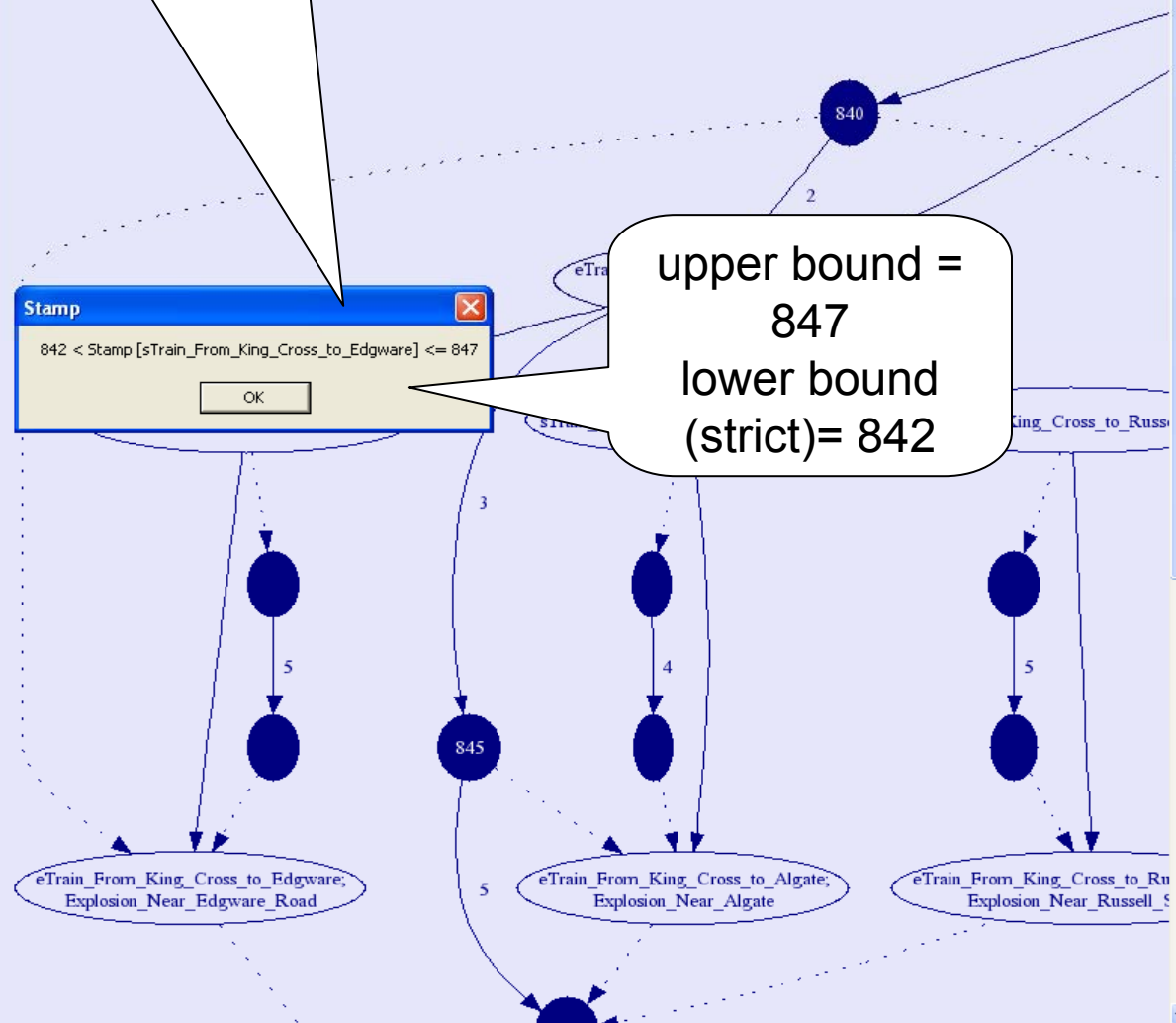
query Stamp (when did the train to Edgware leave from King's Cross?)

Stamp

842 < Stamp [sTrain_From_King_Cross_to_Edgware] <= 847

OK

upper bound = 847
lower bound (strict)= 842



Summary of Approach



- Convert the available temporal information into statements in Point-Interval Logic.
- Input these statements to Temper using the language editor of Temper.
- Construct a Point Graph representation of the set of PIL statements.
- If the set of PIL statements is inconsistent then Temper will not be able to construct the Point Graph representation.
- Temper will identify the subset of PIL statements causing the inconsistency.
- Remove the inconsistent statements.
- Once a consistent Point Graph has been constructed, it can be used to draw inferences.

Questions?